3Doodler Geometry: Pyramids

Suitable Ages:
Suitable for ages 14+

Skill Level:
Basic

Materials Required:
- 3Doodler 2.0: At a minimum, one 3Doodler per group (six groups per class); if more 3Doodlers are available, one for every one to two students is advisable
- 3Doodler PLA or ABS strands: 3 strands per 3Doodler
- Pyramid Stencils (**N.B.: If PLA is used, it is advisable to apply a layer of masking tape over the stencil to ensure that the shapes can be peeled off the stencil with ease**)
- The Pyramid Worksheet (pages 16-19)

Duration:
This classroom activity will take between two and three hours, which can be divided between a number of classroom sessions

Instructional Video:
Instructional videos showing how to build the Pyramids 1 & 4 of this activity are available at:
YouTube: https://youtu.be/f0wQQKNY51w and https://youtu.be/coKS6OJh8WU
Vimeo: https://vimeo.com/140039345 and https://vimeo.com/140039344
Dropbox: https://www.dropbox.com/s/hvpr8mcv6f135yi/Example%20Pyramid%201.mpeg?dl=0 and https://www.dropbox.com/s/xf004dw7o4lzcwq/Example%20Pyramid%204.mpeg?dl=0

Written by: Linda Giampieretti
Part of the "Fascinating World of Geometric Forms" Project

Contents

Objective

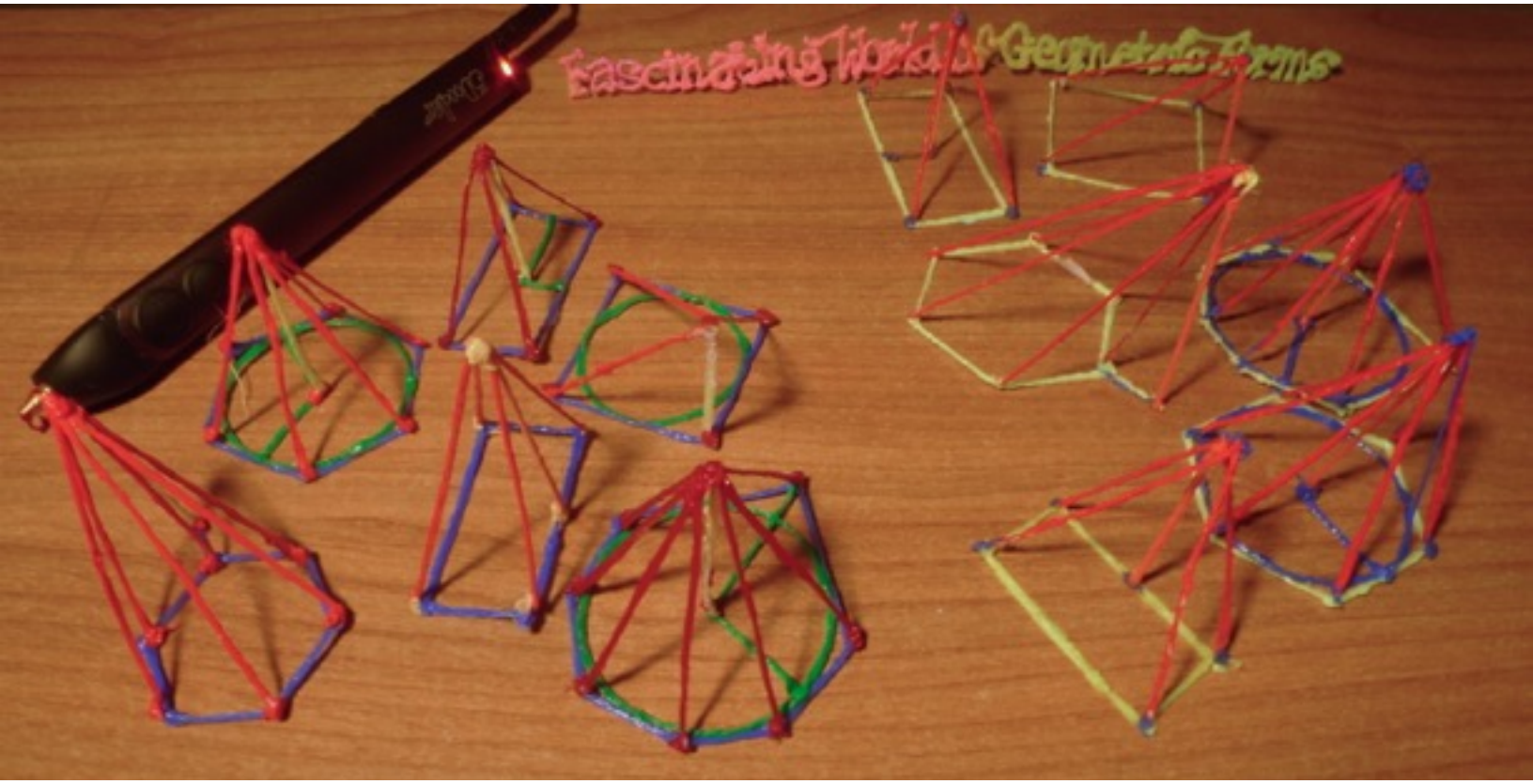
Triangles, and in turn, pyramids, form the basis for much of the structural strength in modern architecture and engineering. This activity explores the various types of pyramids, as well as the polygons that compose them. The activity will provide students with an in-depth understanding of how pyramids are formed, their characteristics, and how they are categorized and analyzed.

On completion of this activity, students will be able to understand the following geometric concepts:

- Definition and characteristics of polygons generally
- Distinction between regular and irregular polygons
- Distinction between convex and concave polygons
- Distinction and characteristics of pyramids generally
- Distinction between right and non-right pyramids

Students will also develop the ability to:

- Visualize three-dimensional geometric figures
- Transition from two-dimensional geometric figures to three-dimensional geometric forms
- Transition back from three-dimensional geometric forms to two-dimensional figures.



Activity

Part 1: Core Concepts

1. With your students, review the following definitions:
   - polygon
   - vertex
   - edge of polygon
   - height of polygon
   - convex and non-convex polygons
   - regular polygon
   - polyhedron
   - face, edge and vertex of a polyhedron.

2. Explain to students the definitions of the **incircle**, **incenter**, and **a polygon circumscribed around a circle** (Definition 1 in the Reference Materials on page 7).

3. For context, explain to students the importance of pyramids, pointing out real-world examples (as used in ancient monuments as well as current architecture and design, i.e. **The Louvre in Paris**, **Transamerica Pyramid in San Francisco**, and the **Inverted Pyramid Building in Bratislava, Slovakia**). Then review the definition of a pyramid (Definition 2 in the Reference Materials, Page 7).

Part 2: Hands On

1. Divide the class into **6 groups** — one for each pyramid stencil included with this classroom activity (pages 10-15).

2. Provide to each group the following materials:
   - 1 × 3Doodler
   - 3 × strands of PLA or ABS plastic
   - 1 × pyramid stencil

3. Each group will be given **45 minutes** to 3Doodle a pyramid using the stencil provided. Give each group the following general instructions:
   - Use the 3Doodler to trace each part of the pyramid.
   - Use different colors of 3Doodler plastic to differentiate the various parts of the pyramid.
   - Peel the individual pieces off of the paper stencil when complete.
   - Join the pieces together correctly with the 3Doodler.

4. Ask each group to describe their pyramid to the rest of the class. Ask them to answer the following questions:

- Which polygon is the base?
- Is the polygon circumscribed around a circle?
- Which polygons are the lateral face?

Part 3: Advanced Concepts + Hands On

1. Review the definitions of a **right pyramid**, **oblique pyramid**, **apothem**, **regular pyramid**, and t**he altitude of pyramid** (Definitions 3, 4, and 5 in the Reference Materials on page 8).

2. Ask to the groups to answer the following questions:
   - Is your pyramid a right pyramid?
   - Is your pyramid oblique?
   - Is your pyramid regular?
   - Does your pyramid have an apothem?

3. Next, ask your students to build a pyramid **without** a stencil. Specify what type of polygon should be used as the base, and whether the pyramid will be a right pyramid or an oblique pyramid.

4. For reference, direct students to this Geogebra resource: http://tube.geogebra.org/m/JT5DUBXu.

Supplement: In-Depth

Exploring the Volume of a Pyramid

There is a special relationship between the volume of a pyramid and the volume of a prism of the same height (each having the same polygon as it's base). The volume of the pyramid will be exactly 1/3 the volume of the prism.

The pyramid from Stencil 3 can be used to demonstrate this in a highly intuitive way.

Have your students make three of the pyramids from Stencil 3 and fit them together to make a cube. The students will see that the volume of each pyramid is one third of the volume of a prism (provided the prism has the same altitude and base of the pyramid).

For reference, see Exercise 3 of this Geogebra resource: http://tube.geogebra.org/m/JT5DUBXu.

Euler Characteristic

The pyramids in this activity can also be used to introduce the Euler characteristic, *X*. The Euler characteristic is a number that is used to classify different types of surfaces, and is expressed in the formula $X = V - E + F$, where:

- V = number of vertices (corners);
- E = number of edges; and
- F = number of faces.

Any convex polyhedron's surface has Euler characteristic $2 = V - E + F$.

Sequence & Pacing

This classroom activity can be organized to stretch across **three or four classroom periods**, or can stand alone as a single three-hour activity.

Striving Students

Paying close attention to these students will ensure safe use of the 3Doodler as well as readily available assistance. Grouping striving students with accelerated students can provide additional support.

Accelerated Students

Grouping these students with striving students will give them an opportunity to provide assistance and encouragement to their fellow classmates, reinforcing the lesson for both parties. Accelerated students who complete their work early should be encouraged to further explore by creating additional three-dimensional geometric forms.

Evaluation Strategies

Students will be graded according to a rubric on their attentiveness and participation as well as the completion of the included hand-out. Successful use of the 3Doodler will also contribute to the student's grade. The hand-out will be collected at the end of class and corrected for use as reference material for future lessons.

Evaluation Rubric

	4	3	2	1
Participation	Students raise their hands to ask and answer pertinent questions. Follow directions. Stay on task.	Students raise their hands to ask and answer pertinent questions. Follow directions.	Students answer when called upon. Follow directions.	Students fail to engage in classroom discussion. Failure to follow directions/stay on task.
Attentiveness	Students are following along and paying close attention at all times.	Students are following along with the lesson.	Students need to be reminded of instruction due to lack of attention.	Students are not following along/ paying attention.
Loading, Extrusion, Reversal, Trimming	Safe and successful operation of the 3Doodler.	Safe and successful operation of the 3Doodler.	Successful operation of the 3Doodler.	Failure to operate the 3Doodler.
Pyramid Worksheet	All blanks are filled in; up to 1 mistake.	All blanks are filled in; up to 3 mistakes.	All blanks filled in; up to 5 mistakes.	Failure to fill in all blanks; more than 5 mistakes.

Reference Materials

Definition 1: incircle, incenter, polygon circumscribed around a circle

In Euclidean geometry, a tangential polygon, also known as a circumscribed polygon, is a convex polygon that contains an **inscribed circle**, the largest circle that will fit inside the polygon and that touches each side of the polygon (also called an **incircle**). This circle is tangental to each of the polygon's sides. The center of the incircle is called the **incenter**.

A polygon is circumscribed around a circle if the bisectors of all the angles intersect at a single point (the center of the circle) which is therefore called the incenter of the polygon. Additionally:

- Triangles are always circumscribed around a circle.
- Quadrilaterals are not always circumscribed around a circle. We can say that a quadrilateral is circumscribed around a circle if, and only if, the sum of the measurements of two opposite sides is equal to the sum of the other two sides.
- Regular polygon are always circumscribed around a circle.
- Polygons with more than 4 sides can be circumscribed around a circle or not. The test for this is to draw the bisectors of all the angles and see if they intersect in a single point or not.

The **apothem** of a polygon is the radius of the incircle. It is the fixed distance from the incenter to each sides of the polygon.

Definition 2: Pyramid

A **pyramid** is a polyhedron formed by connecting a polygonal base and a point not in the plane of the polygon, called the **apex**. Each base edge and apex form a triangle, called a **lateral face**.

The **altitude (or the height)** of a pyramid is the distance from the apex to the base of the pyramid. Formally, the shortest line segment is between the apex of a pyramid and the base. It is the segment from the apex that is orthogonal to the plane that contains the base of the pyramid.

Definition 3: Right Pyramid

A **right pyramid** is a pyramid whose base is a polygon and whose altitude intersects the plane of its base at the incenter of the base. The altitude of each lateral face are all the same length and are called **apothem** of the pyramid; it is because these altitudes are the hypotenuses of right triangles whose legs are always the altitude of the pyramid and the radius of the incircle (the apothem of the base polygon). Right pyramids can be regular or irregular.

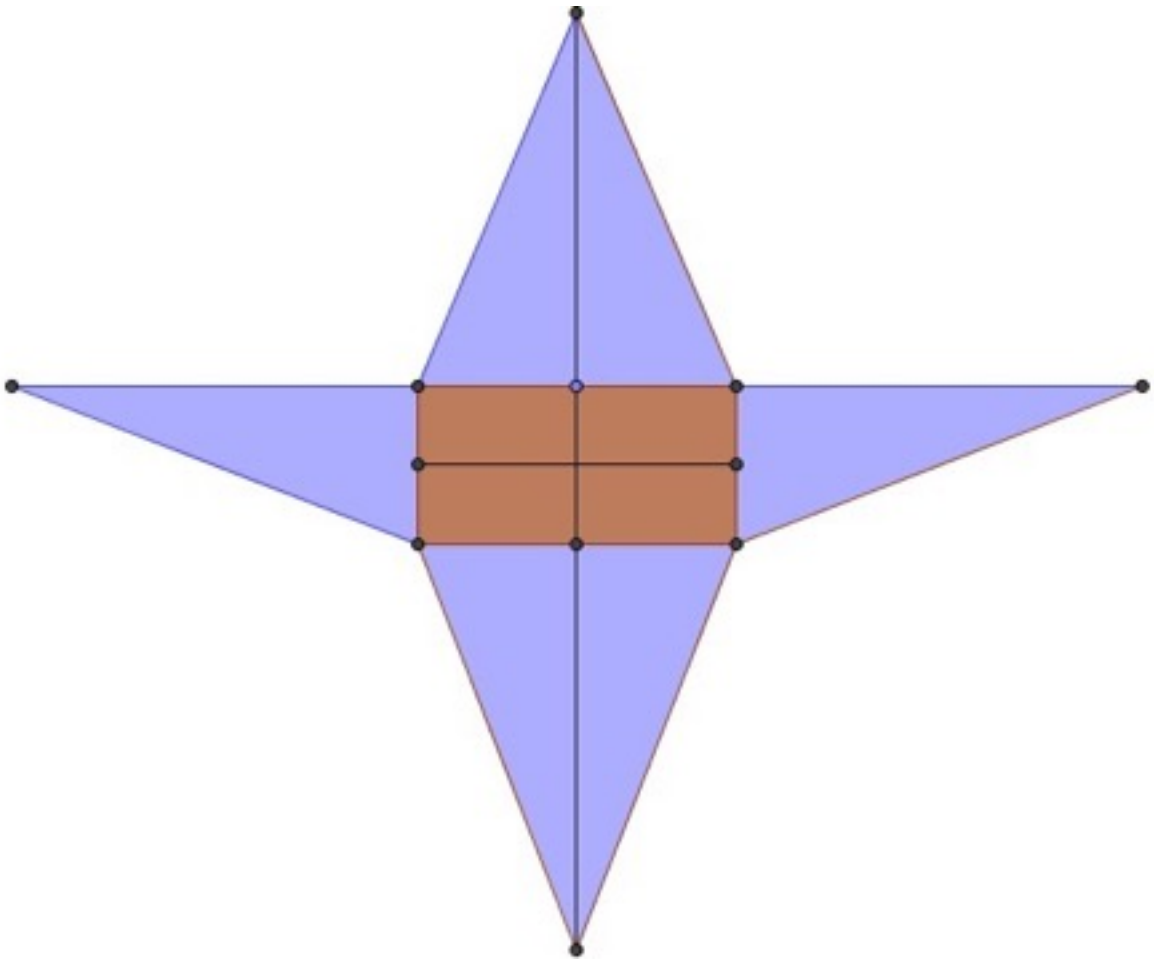
Definition 4: Oblique Pyramid

Pyramids that are not right pyramids are called **oblique pyramids**. We have an oblique pyramid if **the altitude of the pyramid does not intersect the plane of its base at the incenter of the base.** An exception to this is if the base is an irregular polygon, then the base usually has no defined center point.

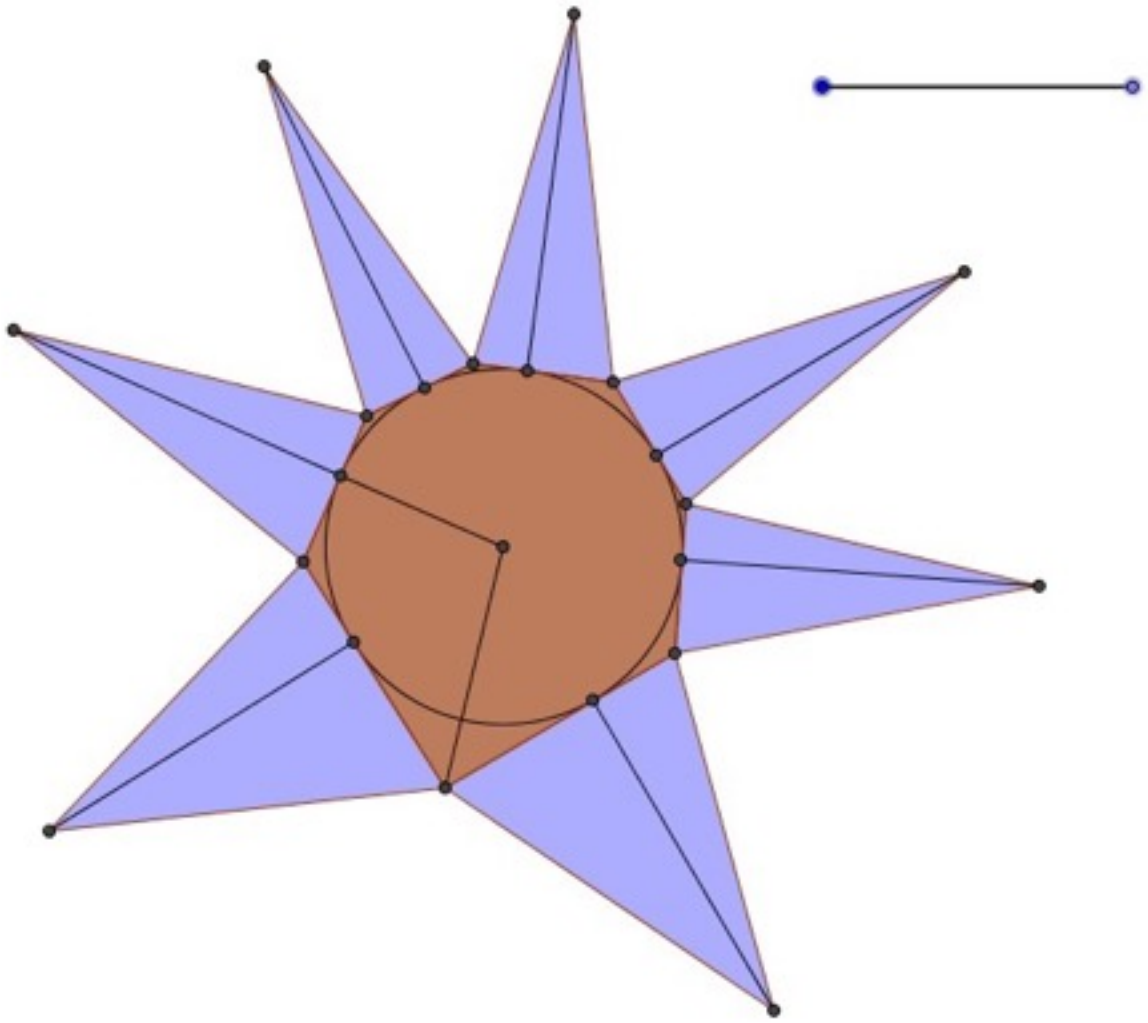
Definition 5: Regular Pyramid

A **regular pyramid** is a pyramid whose base is a regular polygon (polygon circumscribed around a circle) whose altitude intersects the plane of its base at the center of the base (the incenter). Regular pyramids are also **right pyramid** (however, the contrary is not always true, i.e. not all right pyramids are regular pyramids!). The altitudes of each lateral face are all the same length and are called **apothem** of the pyramid. Furthermore, only a regular pyramid has all lateral faces congruent.

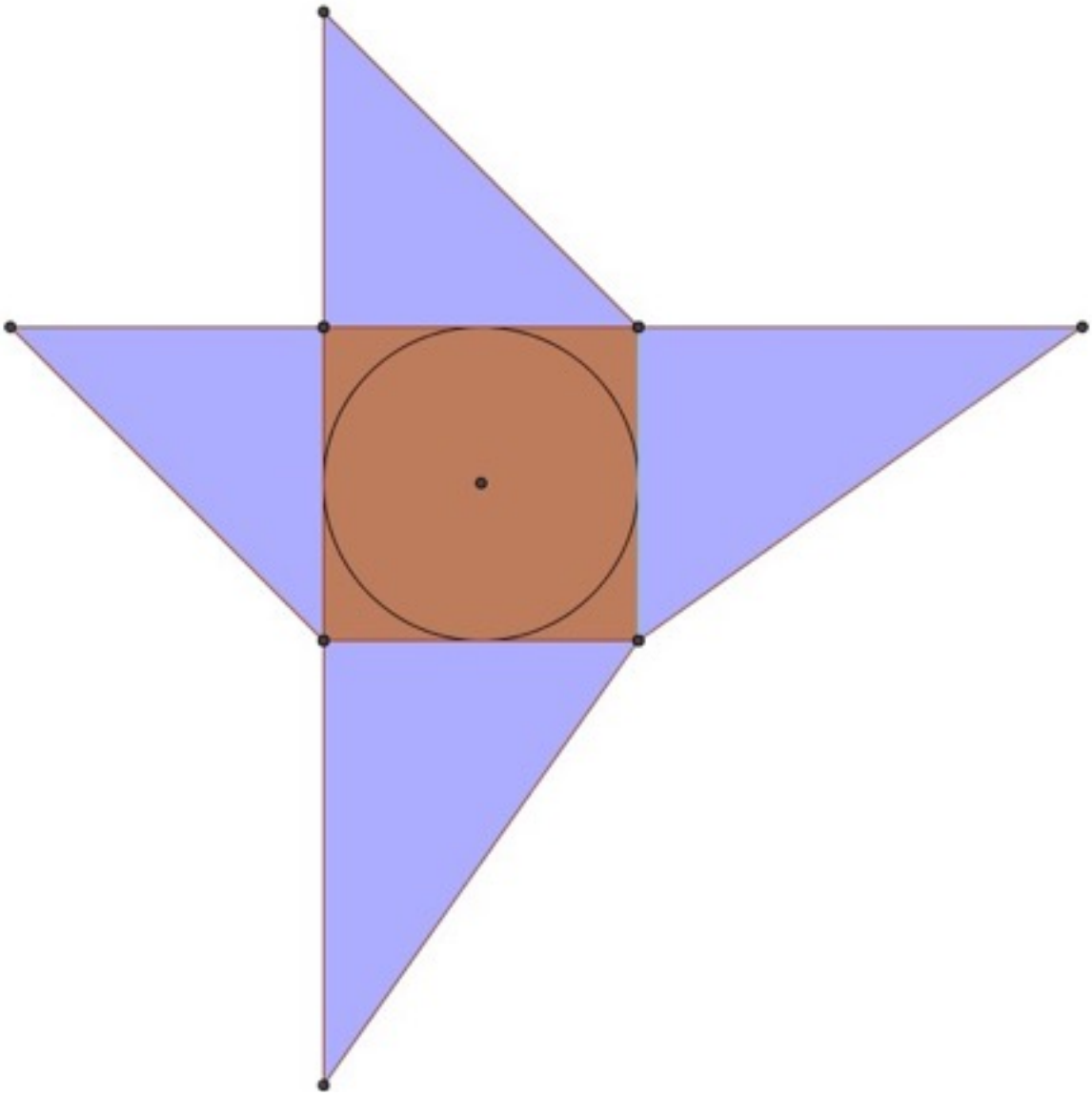
Stencil: Pyramid 1



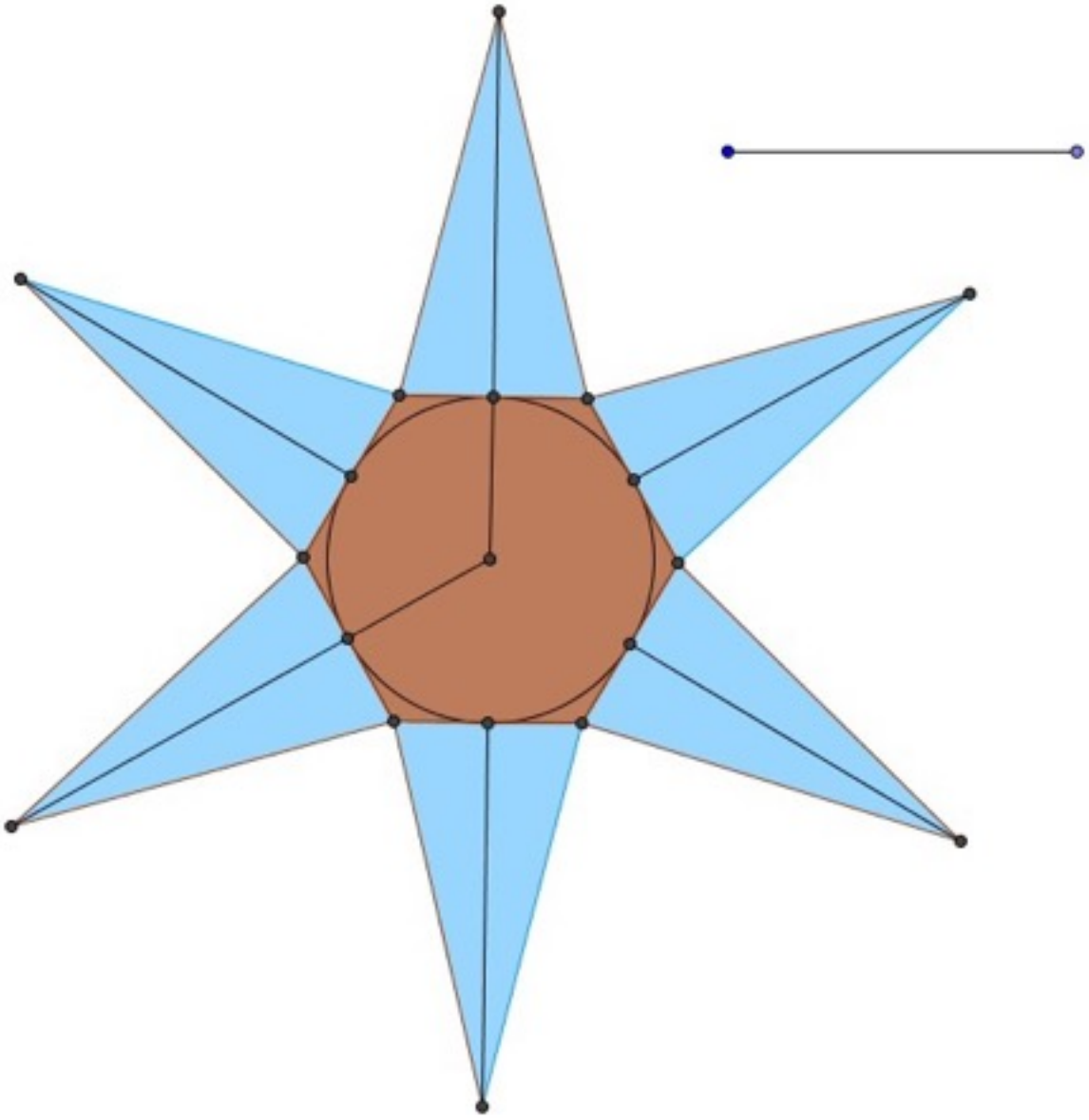
Stencil: Pyramid 2



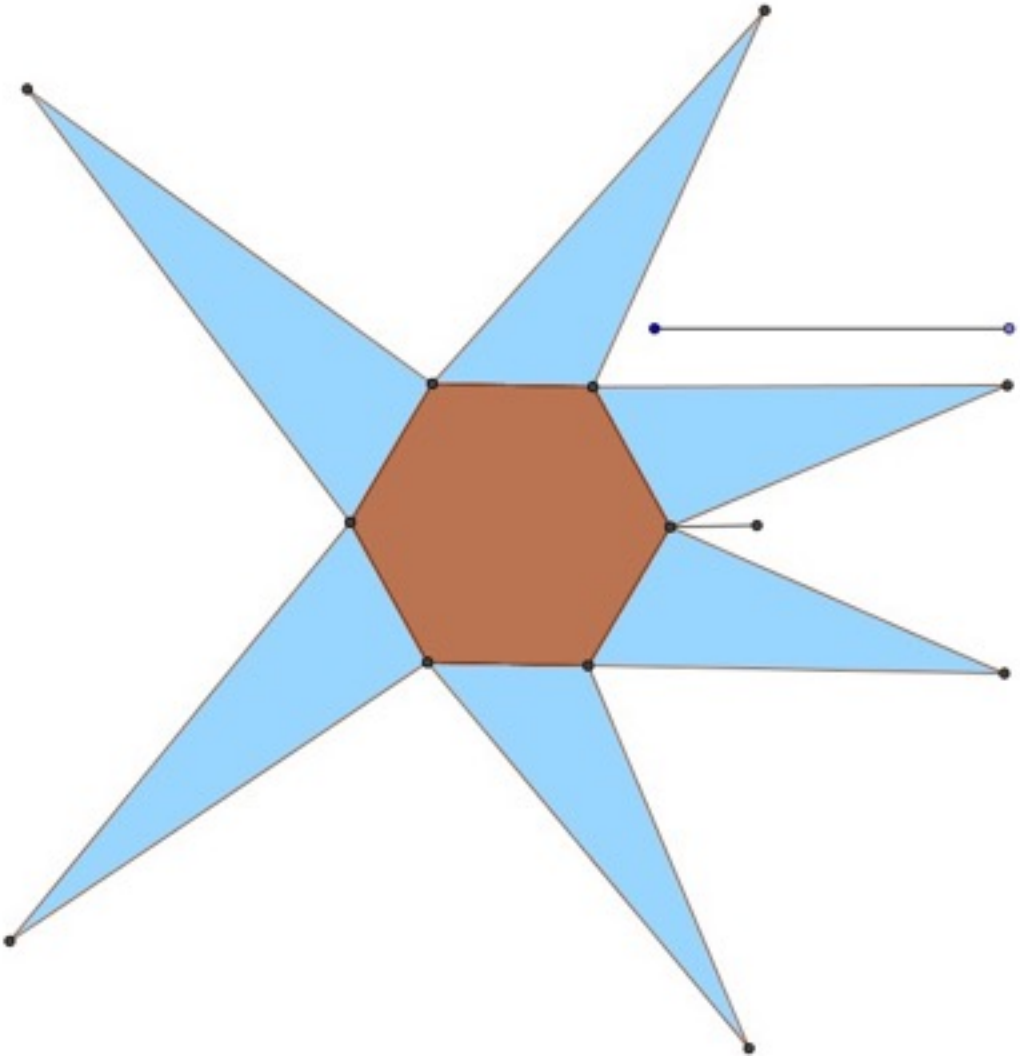
Stencil: Pyramid 3



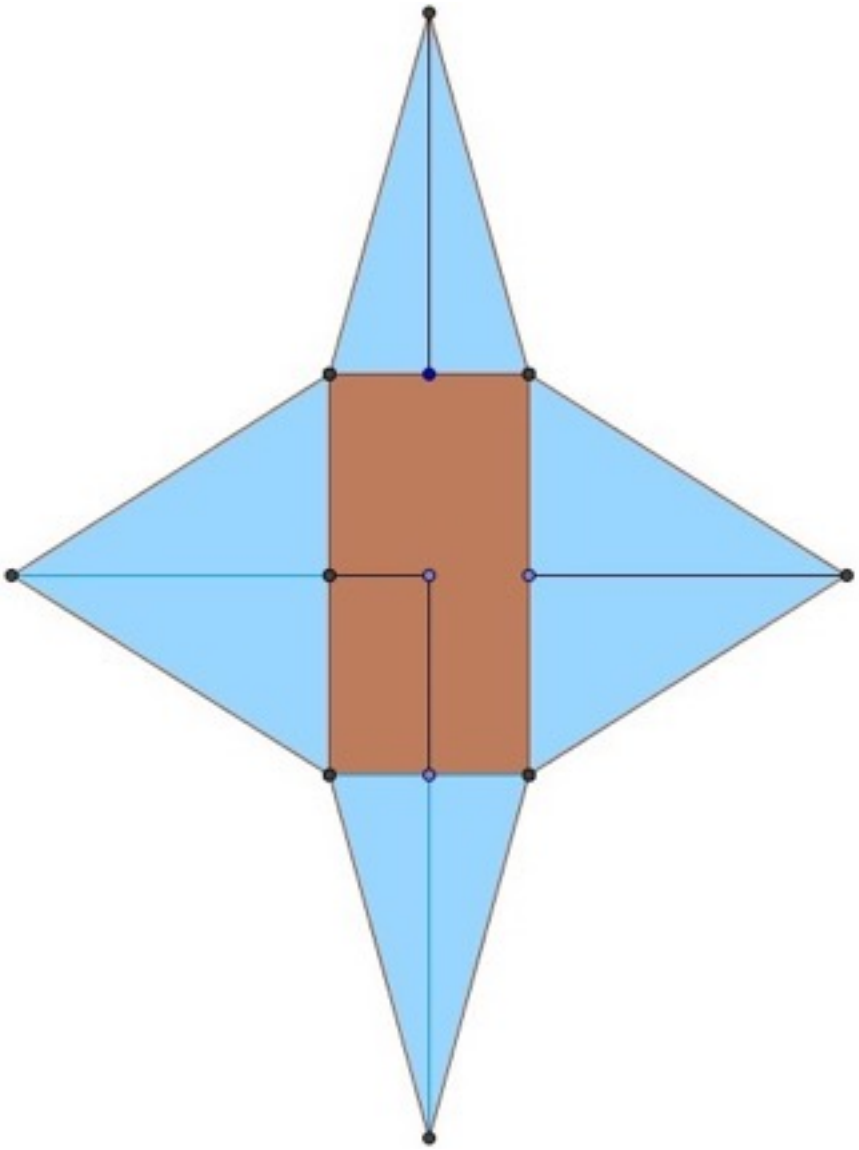
Stencil: Pyramid 4



Stencil: Pyramid 5



Stencil: Pyramid 6



Pyramid Worksheet

Pyramid 1

Question	Answer
Which polygon is the base?	
Is the polygon circumscribed around a circle?	
Which polygons make up the lateral face?	
Is this a right pyramid?	
Is this an oblique pyramid?	
Is this a regular pyramid?	
Does this pyramid have an apothem?	

Pyramid 2

Question	Answer
Which polygon is the base?	
Is the polygon circumscribed around a circle?	
Which polygons make up the lateral face?	
Is this a right pyramid?	
Is this an oblique pyramid?	
Is this a regular pyramid?	
Does this pyramid have an apothem?	

Pyramid 3

Question	Answer
Which polygon is the base?	
Is the polygon circumscribed around a circle?	
Which polygons make up the lateral face?	
Is this a right pyramid?	
Is this an oblique pyramid?	
Is this a regular pyramid?	
Does this pyramid have an apothem?	

Pyramid 4

Question	Answer
Which polygon is the base?	
Is the polygon circumscribed around a circle?	
Which polygons make up the lateral face?	
Is this a right pyramid?	
Is this an oblique pyramid?	
Is this a regular pyramid?	
Does this pyramid have an apothem?	

Pyramid 5

Question	Answer
Which polygon is the base?	
Is the polygon circumscribed around a circle?	
Which polygons make up the lateral face?	
Is this a right pyramid?	
Is this an oblique pyramid?	
Is this a regular pyramid?	
Does this pyramid have an apothem?	

Pyramid 6

Question	Answer
Which polygon is the base?	
Is the polygon circumscribed around a circle?	
Which polygons make up the lateral face?	
Is this a right pyramid?	
Is this an oblique pyramid?	
Is this a regular pyramid?	
Does this pyramid have an apothem?	

Worksheet Answer Key

Pyramid 1

Question	Answer
Which polygon is the base?	Rectangle
Is the polygon circumscribed around a circle?	No
Which polygons make up the lateral face?	Two different isosceles triangles and two right triangles
Is this a right pyramid?	No
Is this an oblique pyramid?	Yes
Is this a regular pyramid?	No
Does this pyramid have an apothem?	No

Pyramid 2

Question	Answer
Which polygon is the base?	Irregular polygon of seven sides
Is the polygon circumscribed around a circle?	Yes
Which polygons make up the lateral face?	Seven different scalene triangles
Is this a right pyramid?	Yes
Is this an oblique pyramid?	No
Is this a regular pyramid?	No
Does this pyramid have an apothem?	Yes

Pyramid 3

Question	Answer
Which polygon is the base?	Square
Is the polygon circumscribed around a circle?	Yes
Which polygons make up the lateral face?	Right triangles
Is this a right pyramid?	No
Is this an oblique pyramid?	Yes
Is this a regular pyramid?	No
Does this pyramid have an apothem?	No

Pyramid 4

Question	Answer
Which polygon is the base?	Hexagon
Is the polygon circumscribed around a circle?	Yes
Which polygons make up the lateral face?	Six congruent isosceles triangles
Is this a right pyramid?	Yes
Is this an oblique pyramid?	No
Is this a regular pyramid?	Yes
Does this pyramid have an apothem?	Yes

Pyramid 5

Question	Answer
Which polygon is the base?	Hexagon
Is the polygon circumscribed around a circle?	Yes
Which polygons make up the lateral face?	All different scalene triangles
Is this a right pyramid?	No
Is this an oblique pyramid?	Yes
Is this a regular pyramid?	No
Does this pyramid have an apothem?	No

Pyramid 6

Question	Answer
Which polygon is the base?	Rectangle
Is the polygon circumscribed around a circle?	No
Which polygons make up the lateral face?	Opposite faces are congruent Isosceles triangles
Is this a right pyramid?	No
Is this an oblique pyramid?	No
Is this a regular pyramid?	No
Does this pyramid have an apothem?	No

Additional Resources

Instructional Video

Instructional videos showing how to build the Pyramids 1 & 4 of this activity are available at:
YouTube: https://youtu.be/f0wQQKNY51w and https://youtu.be/coKS6OJh8WU
Vimeo: https://vimeo.com/140039345 and https://vimeo.com/140039344
Dropbox: https://www.dropbox.com/s/hvpr8mcv6f135yi/Example%20Pyramid%201.mpeg?dl=0 and https://www.dropbox.com/s/xf004dw7o4lzcwq/Example%20Pyramid%204.mpeg?dl=0

Online Geometry Resources

Geogebra

All pyramids used for this classroom activity are included in this Geogebra page: http://tube.geogebra.org/m/JT5DUBXu

Math Open Reference

- Polygon: http://www.mathopenref.com/polygon.html
- Convex polygon: http://www.mathopenref.com/polygonconvex.html
- Incircle of a polygon: http://www.mathopenref.com/polygonincircle.html
- Apothem of a polygon: http://www.mathopenref.com/apothem.html
- Pyramid: http://www.mathopenref.com/pyramid.html
- Volume of a pyramid: http://www.mathopenref.com/pyramidvolume.html

Wikipedia

- Pyramid: https://en.wikipedia.org/wiki/Pyramid_(geometry)
- Polygon: https://en.wikipedia.org/wiki/Polygon
- Euler Characteristic: https://en.wikipedia.org/wiki/Euler_characteristic

Other

- Hotmath: Pyramid: http://hotmath.com/hotmath_help/topics/pyramid.html
- Inscribed and circumscribed polygons: http://www.bymath.com/studyguide/geo/sec/geo11.htm
- Right pyramid: http://www.mathwords.com/r/right_pyramid.htm
- Apothem of a pyramid: http://www.ditutor.com/solid_gometry/apothem_pyramid.html

Fascinating World of Geometric Forms

Continue your exploration of geometric forms with Linda Giampieretti: https://www.youtube.com/channel/UCWV8EmazidHZgYNxHItPDKA or https://www.facebook.com/Fascinating-World-of-Geometric-Forms-502595993215052/timeline/?ref=bookmarks.

Tutorial Videos

Please visit the3Doodler.com/videos/#started to find videos demonstrating the skills required for this activity. Individual videos that will be useful include:

- Inserting Plastic:
  - YouTube: https://www.youtube.com/watch?v=ZSmdhZEnMDE
  - Dropbox: https://www.dropbox.com/s/3jnmafuve2saqu4/Inserting%20Plastic.mp4?dl=0
- The Buttons:
  - YouTube: https://www.youtube.com/watch?v=mos2SBukObo
  - Dropbox: https://www.dropbox.com/s/cqkozrmhktr3u38/Buttons.mp4?dl=0
- Reversing Plastic:
  - YouTube: https://www.youtube.com/watch?v=aD84E55mgac
  - Dropbox: https://www.dropbox.com/s/mpzxcrky9f5aq41/Reversing%20Plastic.mp4?dl=0

Additional Inspiration

For additional inspiration and ideas about other simple projects that can be accomplished at this level, check out the following links:

- Stencils and Projects: the3Doodler.com/community/
- Doodles by You: the3Doodler.com/doodles/
- Videos: the3Doodler.com/videos/
  - Getting Started: the3Doodler.com/videos/#started
  - Tips & Tricks: the3Doodler.com/videos/#tips

3Doodler/EDU

More curricular materials are available at the3Doodler.com/education/.

If you have additional ideas for classroom activities or lessons, feel free to reach out to us at education@the3Doodler.com!

Special Thanks

The 3Doodler team would like to thank Linda Giampieretti for her pioneering work in geometry education with the 3Doodler. Her passion for engaging and fascinating young minds with her research and project work, “Fascinating World of Geometric Forms”, is an inspiration to us all.

You can find Linda on Twitter (@LindaGiamp) and through her YouTube page (https://www.youtube.com/channel/UCWV8EmazidHZgYNxHItPDKA).